THE MATHEMATICAL ASSOCIATION OF VICTORIA

THE COMMON DENOMINATOR

1/20

METACOGNITION

INSIDE

Paul Tarabay - Learning and teaching leader, St. Antonine College

As an educator, I believe effective mathematics teaching requires understanding what students know and need to learn, and then challenging and supporting them to learn it well. On a daily basis, educators describe and explain why we are aiming for particular mathematical goals before teaching through learning intentions and success criteria. The challenge for teachers and schools is to develop a shared understanding of what excellent practice looks like. While it will not look exactly the same in every classroom, there are some instructional practices that evidence suggests work well in most applications, such as HITS (Department of Education, June 2017).

The high impact teaching strategies (HITS) are 10 instructional practices that reliably increase student learning when they're applied. Metacognition is one of the high-impact teaching strategies that challenges students' thinking, and their thinking about thinking.

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THE COMMON DENOMINATOR
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The Mathematical Association of Victoria, 61 Blyth Street
Brunswick VIC 3056

ABN: 34 004 892 755
Tel: 03 9380 2399
Fax: 03 9389 0399

office@mav.vic.edu.au
www.mav.vic.edu.au

Office hours:
8.30am – 5pm
Monday to Friday

President: Michael O'Connor
CEO: Peter Saffin

The Common Denominator is edited and produced by Louise Gray, Stitch Marketing.

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FROM THE PRESIDENT

Michael O'Connor

Welcome back to the start of a new year. Last December, MAV's annual conference was themed *Making Connections*. Some 1400 teachers participated in nearly 300 different sessions including eleven keynotes. Two days before the conference began, the latest in the series of PISA results were released amid much handwringing and comment from many in politics and the media. So, naturally, this became part of the conversation for speakers and delegates alike.

Over the period from 2003 when the first PISA results were released, Australia has consistently slipped in the rankings. While it is simplistic in the extreme to take test scores as the sole measure of success we do have to acknowledge there is room for improvement. Some good news is that in mathematics, Victoria still outperformed the majority of other Australian jurisdictions and had the lowest level of decline of all states and territories.

In his keynote, Professor Mike Askew noted that the items in which Australian students were weakest in on PISA were not the basics of arithmetic but in making connections through skills of reasoning. This was not intended as an argument for either/or reform of the curriculum but rather both/and as a number of his examples went on to demonstrate.

A number of the other keynotes also grappled with how we, as a community of teachers of mathematics, can best serve our students. The issues are complex and wide ranging. It is widely agreed that there are no easy answers or quick fixes.

As the attendance at the conference bore witness to, you are passionate, engaged and committed to providing the best education for your students that you can. As teachers, and providers of professional learning and support to teachers, your needs and concerns for providing high quality instruction to students should be foremost in the minds of policy makers. For this to happen beyond the headlines and hype, the conversation has to continue. Your stories of what has worked need to be shared and celebrated. The MAV at state level, and the AAMT nationally, are your voice to government and industry. So, I take this opportunity to encourage you, individually, as a faculty, or as a whole school, to share your successes and your challenges with us so that we can pass them on.

LIFE MEMBERS - 2019

In December 2019 the MAV Board and members welcomed three new life members. The criteria for life membership is rigorous and is focused around:

- Long term involvement and support of the MAV
- Significant and long term-contribution to mathematics education in Victoria
- Significant and long term-contribution to valuing mathematics in society

Congratulations to Dr Ian Lowe, Professor Colleen Vale and Emeritus Professor Peter Sullivan for having the honour of life membership bestowed upon them.

IAN LOWE

Through his long career as a teacher/teacher educator and his association with the MAV, Ian has been committed to supporting teachers in the classroom, in fostering high quality teaching and in expanding the horizons of mathematics teaching and learning, especially for students for whom mathematics is needed to support other school subjects and to keep open prospects for further education and training.

Ian was a professional officer at MAV from 2005 to 2017. During this time, he wrote many books and resources that were freely published through MAV. Many of you may remember *RIME* (*Reality in Maths Education*), which is still widely recognised for its engaging activities. Ian also ran extensive professional development programs and supported schools with various needs. Ian has been a teacher, lecturer, and a mentor and support to many new mathematics educators.

MAV PROFESSIONAL DEVELOPMENT

During Term 1 2020, a variety of presenters and MAV's own mathematics educational consultants will present workshops focusing on innovative teaching practice.

Make sure you reserve a place by booking online early, www.mav.vic.edu.au/pd.

TOPIC	DATE	YEARS	PRESENTER
VCE Mathematics day out - Melbourne University	14/2/20	VCE	Various
VCE Mathematics day out - Federation University, Gippsland	17/2/20	VCE	Various
SAC workshops - Burwood (Specialist and Further)	25/2/20	VCE	Various
SAC workshops - Burwood (Methods)	TBC	VCE	Various
VCE Mathematics day out - La Trobe University, Bendigo	28/2/20	VCE	Various
Online interactive: Digital learning (mathematics and digital technology)	2/3/20	F - 6	DLTV
Sunraysia conference	10/3/20	F - 10	Various
Meet the Assessors - Wangaratta	10/3/20	VCE	Various
Meet the Assessors - Geelong	11/3/20	VCE	Various
Online interactive: Digital learning	11/3/20	F - 6	Paul Swan
Mathematics proficiencies: holding our maths curriculum together	12/3/20	F - 8	Paul Swan
SAC workshops - Hoppers Crossing	12/3/20	VCE	Various
Meet the Assessors - Williamstown	16/3/20	VCE	Various
Meet the Assessors - Horsham	17/3/20	VCE	Various
SAC workshops - Terang	23/3/20	VCE	Various
Meet the Assessors - Burwood (Specialist and Further)	24/3/20	VCE	Various
Meet the Assessors - Burwood (Methods)	25/3/20	VCE	Various

PETER SULLIVAN

Peter is well-known for his extensive work in initial teacher education, teacher professional learning and mathematics education research. At all times his career goal has been to improve the teaching and learning of mathematics. His research has had a broad impact from early childhood education to senior secondary mathematics through an extensive publication record, presentations at local, state, national and international conferences and his influential contribution to many significant committees and review panels. From 2005 to 2008, Peter was a member of the Social, Behavioural and Economic Sciences panel of the Australian Research Council College of Experts. He has also worked as Editor for the *Journal of Mathematics Teacher Education* and the *Mathematics Education Research Journal*. Peter is a past President of AAMT, and a past council member of the MAV. He was the lead writer of the *Australian Curriculum: Mathematics*. In 2016 Peter received the Career Research Medal.

Peter continues to support MAV, running professional learning sessions and attending various events.

COLLEEN VALE

Colleen has a long and significant history in mathematics education across both primary and secondary sectors, including as a secondary school teacher, and in relation to teacher training, research, publishing and professional development. She has been an MAV Council member, including being President of the MAV, and has been a regular presenter at MAV conferences. Colleen is co-author of the award winning book *Teaching Secondary School Mathematics: Research and Practice for the 21st Century* and is well known for her professional learning programs with out-of-field junior secondary mathematics teachers, research and projects with teachers on the use of technology and for her interest and work in the field of social justice in mathematics education.

METACOGNITION:
A HIGH IMPACT TEACHING STRATEGY

Paul Tarabay - Learning and teaching leader, St. Antonine College

CONT. FROM PAGE 1.

If our intention is to maximise the learning of our students, then it is necessary to inform our decisions by delving into the research in the field of mathematics education. In 2016, I completed my postgraduate studies at Monash University. I submitted my research paper on metacognition for my final assessment. During the course of my study, I investigated the role of problem solving as an instructional focus to enhance students' mathematical understanding and their general numeracy learning outcomes. In this article I will share my understanding of metacognition and its implication on teaching and learning.

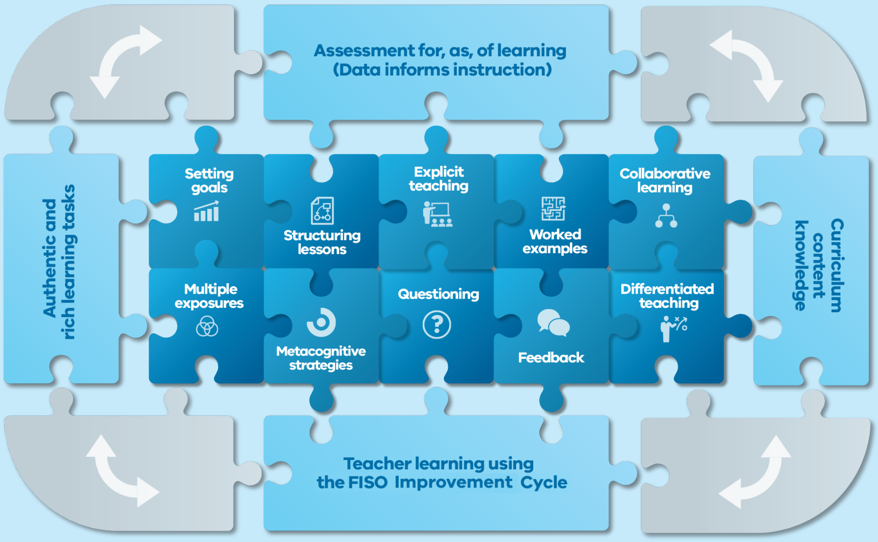

Image source: Department of Education, June 2017.

WHAT IS METACOGNITION?

Wilson & Clarke (2004) defined metacognition as the awareness that individuals have of their own thinking, their evaluation of that thinking; and their regulation of that thinking. Curriculum writers nationally and internationally have highlighted the importance of metacognition in mathematics education. According to the VCAA, 'Learning is enhanced when individuals develop the capacity to reflect on and refine their existing ideas and beliefs. In this dimension students learn to reflect on what they know and develop awareness that there is more to know... They also develop their metacognitive skills in planning, monitoring and evaluating their own thinking processes and strategies'.

According to Flavell (1976) the range of metacognitive skills we have and use, varies from individual to individual and from context to context. Figure 1 illustrates Flavell's model of metacognition that it is composed of four classes of variables.

1. Metacognitive knowledge refers to what learners know about learning. This includes:

- the learner's knowledge of their own cognitive abilities: e.g. 'I have trouble remembering names of 2D shapes.'
- the learner's knowledge of particular tasks: e.g. 'I will get the area of a rectangle when I multiply length by width.'
- the learner's knowledge of different strategies that are available to them and when they are appropriate to the task: e.g. 'I will be able to solve a particular problem if I use the working backward strategy.'

2. Metacognitive strategies refer to the actions learners take during the learning process. Metacognitive strategies involve planning, monitoring, evaluating and reflecting (Krause, 2003).

During the *planning* phase, learners think about the learning goal, how they will approach the task and which strategies will use. At this stage, it is helpful for learners to ask themselves:

- What am I being asked to do? What do I know about this topic?
- Which strategies will I use? What strategies worked last time?
- Have I encountered a question like this before?
- Are there any strategies that I have used before that might be useful?

During the *monitoring* phase, learners implement their plan and monitor the progress they are making towards their learning goal. Students might decide to make changes to the strategies they are using if these are not working. As students work through the task, it will help them to ask themselves:

- How am I doing? Am I on the right track?
- Is the strategy that I am using working?
- How should I proceed? What information is important to remember?
- Should I move in a different direction? Should I adjust the pace because of the difficulty?
- Do I need to try something different? What can I do if I do not understand?

During the *evaluation* phase, students determine how successful the strategy they used was in helping them to achieve their learning goal. To promote evaluation, students could consider:

- How well did I do? What didn't go well?
- What did I learn? Did I get the results I expected?
- What could I have done differently?
- Can I apply this way of thinking to other problems or situations?
- Is there anything I don't understand—any gaps in my knowledge? Do I need to go back through the task to fill in any gaps in understanding?
- How might I apply this line of thinking to other problems?
- What could I do differently next time?
- What went well? What other types of problem can I use this strategy for?

Reflection is a fundamental part of the plan-monitor-evaluate process. Encouraging learners to self-question throughout the process will support this reflection.

3. Metacognitive goals and tasks are key components to metacognition in the planning stage before a task. One such metacognitive question, 'what do I want to achieve?' Goal setting, can help improve performance by focusing attention, enhancing effort and increasing persistence.

4. Metacognitive experiences refer to conscious affective normally accompany monitoring and self-regulation that take place during cognitive activities such as problem solving or reading.

WHY TEACH METACOGNITIVE STRATEGIES?

The importance of metacognitive strategies in schooling has been emphasised in recent curriculum. According to AusVELs, all students will need to develop a set of generic skills that include cognitive and metacognitive or thinking skills. Therefore, as educators we should aim to help our students develop a repertoire of metacognitive strategies to choose from during learning activities. It is well documented that metacognitive strategies:

- Help during problem solving sessions that necessitates more complex and flexible thought processes.
- Are an important element for effective learning outcomes (Campione, J. 1987).
- Have been shown to play an important role in oral communication of information, oral comprehension, reading comprehension, and problem solving (Sternberg, 1990).
- Play an important role in deciding failure or success in mathematical problem solving and tackling challenging tasks. Silver (1985) stated that students having difficulties in mathematics do not use a range of cognitive or metacognitive strategies.
- Have educational implications. Teaching interventions using metacognition have produced higher achievement in mathematical problem solving.
- Empower students to think about their own thinking. Rowe (1988) stated that awareness of the learning process enhances control over students own learning.
- Enhance personal capacity for self-regulation and managing one's own motivation for learning.
- With self-regulation approaches aim to help learners think about their own learning more explicitly.
- Require students to take greater responsibility for their learning and develop their understanding of what is required to succeed.
- Taught explicitly, can help students develop critical and creative thinking skills.
- With self-regulation strategies can lead students to become animated learners.

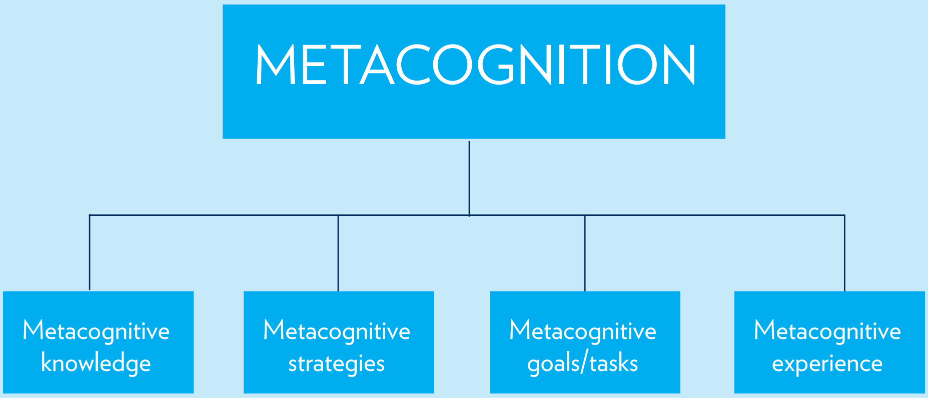

Figure 1. Flavell's model of metacognition.

HOW DO I ENCOURAGE METACOGNITION IN MY CLASS?

I believe it is important to create a classroom culture where it is okay to make mistakes, John Hattie says 'mistakes are the essence of learning'. I often say to my students 'mistakes that we make are the stepping stones to our success!'

Modelling my own mistakes is very helpful to show that it is quite acceptable to make a mistake in the classroom. Furthermore, it is essential for a teacher to develop a supportive classroom environment that allows for metacognitive talk. Talking out loud can help learners to focus and monitor their cognitive processing as well as helping them to develop a deeper understanding of their own thinking processes.

I invite my students to share their thought processes, strategies and ideas in a supportive classroom. During explicit teaching episodes, I model to my students how to question before the task, during the task and after the task.

Clear learning goals are necessary for students to effectively apply their metacognitive strategies.

With clear learning goals, students can plan strategies that will help them to achieve the goals and will also monitor their progress towards achieving these goals.

Keeping a thinking journal can be a highly effective way for learners to develop their ability to plan, monitor and self-evaluate. A thinking journal is a powerful active learning tool that helps students to reflect on how they think. In addition, a thinking journal can encourage a learner to explore, question, connect ideas and persist with their learning.

Another area that helps with the classroom environment, and creating an environment that promotes metacognition, is allowing plenty of time for discussion of strategies. While teaching, I vocalise my internal thoughts to model the use of metacognitive strategies. 'So, what strategies can I use here? What did I do ... what strategies did I use last time when I did a similar activity? Can I think of anything I have done before that might be helpful?'

CHALLENGING TASKS

I teach numeracy through challenging tasks. Challenging tasks require students to plan their approach, especially sequencing more than one step and to process multiple pieces of information, with an expectation that they make connections between those pieces and see concepts in new ways.

In addition, students choose their own strategies, goals, and level of accessing the task and spend time on the task and record their thinking, explain their strategies and justify their thinking to the teacher and other students' (Sullivan, 2015).

METACOGNITION:
A HIGH IMPACT TEACHING STRATEGY (CONT.)

I encourage discussion amongst the students, so group work is really helpful. Also devoting time to planning. When I introduce a new task, I may begin by saying 'So how are we going to tackle this? What do we need to think about?' In doing so, I am encouraging my students to plan and think about strategies in advance of the activity.

THE METACOGNITIVE CLASSROOM

A successful metacognitive classroom is one that looks, very much like an interactive classroom. So, there is a lot of talk going on, it is a classroom where students are all the time looking at what they have done, self-assessing what they have done, and talking about what they have done.

If you went into this classroom you'd hear students verbalising what they are doing in terms of meeting a learning outcome, the strategies that they are following, and whether or not they are working.

The goal of teaching metacognitive strategies is to help learners become comfortable with these strategies so that they employ them automatically to learning tasks, focusing their attention, deriving meaning, and making adjustments if something goes wrong. They do not think about these skills while performing them but, if asked what they are doing, they can usually accurately describe their metacognitive processes.

Because metacognition plays a critical role in successful learning, it is imperative that schools and teachers help learners to develop essential metacognitive skills in order to become successful and independent problem solvers.

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MAVCON SUNRAYSIA

Tuesday 10 March, 2020, The Lake Primary School

Session	Title	
Registration: 8.30am-9am		
Keynote 9am–10am	**F - 10 Reasoning: the glue that holds the curriculum together**	Paul Swan
Session 1 10.05am–11am **Showcase school: Red Cliffs Secondary College**	**1 - 4** Fluency: the facts	Paul Swan
	F - 6 How long is a piece of string?	Jennifer Bowden
	F - 6 Sunraysia Showcase: The catalyst for change Session sponsor: EssentialAssessment Assessment and Curriculum made easy Australian Curriculum • NSW Syllabus • Victorian Curriculum	John Warburton. Tracy Craig and Brooke Allen
	F - 10 Setting 2020 up for success: a session for maths leaders	Ellen Corovic
	7 - 10 Maths for families at the home	Helen Haralambous
	7 - 10 For the love of maths	Tom Moore
Morning tea and networking: 11am–11.30am		
Session 2 11.30am–12.25pm **Showcase school: Merbein P-10 College**	**F - 2** Number talks	Ellen Corovic
	1 - 6 Fluency: the facts	Paul Swan
	F - 6 Step 1, step 2 .... what next?	Judy Gregg
	F - 6 Sunraysia Showcae: Changing the culture	Sue Gardiner
	5 - 8 Differentiation in maths	Tom Moore
	7 - 10 Understanding order of operations	Helen Haralambous
Session 3 12.30pm–1.25pm	**F - 2** Step 1, step 2 .... what next?	Judy Gregg
	F - 6 Number talks	Ellen Corovic
	F - 6 What are the big ideas in geometry?	Jennifer Bowden
	3 - 7 The literacies of mathematics: the missing link	Paul Swan
	7 - 10 Differentiation in maths	Tom Moore
	7 - 10 Engaging students in learning - alternatives to textbook	Helen Haralambous
Lunch and networking: 1.30pm–2.15pm		
Session 4 2.15pm–3.10pm	**F - 2** The importance of play in creative and critical thinking	Jennifer Bowden
	F - 6 Divide and conquer	Judy Gregg
	F - 4 Picture this: using picture books to inspire mathematical thinking	Ellen Corovic
	3 - 7 The literacies of mathematics: the missing link	Paul Swan
	7 - 10 Incorporating technology into your mathematics class	Helen Haralambous and Tom Moore
Closing: 3.15pm-3.30pm		

SHIFTING PERCEPTIONS OF MATHS

Tim Vagg - Numeracy leader, Holy Family Primary School, Bell Park

'I'm not a maths person, I didn't like maths at school, and maths was boring for me'. These are just some of the statements that parents have made in relation to their own experience of maths.

For this mindset to change, we need to immerse parents in environments where they can see that learning maths is fun, is engaging and most of all we all can be problem solvers.

The ideal environment for this to happen is a Family Maths Night. In our planning we wanted to shift our parents thoughts towards maths to a more positive outlook and approach. To help support the shift in mindset and awareness, we opened up our classrooms for families to visit and participate in various games and activities.

FAMILY MATHS NIGHT GOALS

Relevant and engaging
Our students needed to drive the discussion with their parents, to enable this teachers selected games and activities that students were confident and familiar with. When families entered a room, automatically students became the teacher and leader, instructing parents and their siblings on how to play the game, sharing tricks and strategies they have discovered in class. Students showed a passion and enjoyment in sharing their knowledge.

Awareness and understanding
In our planning we discussed the importance of communicating and sharing understanding when solving problems. We wanted parents to see the value of every day tools such as playing cards, dice, dominoes, scales. Objects that are in our homes and are often forgotten about. The Family Maths Night was the perfect setting to reintroduce parents to such games as *Race to 100* with dice, *Place Value Rocket Numbers*, making the biggest and smallest numbers with cards, the list is endless.

The discussion and excitement that was happening with families was terrific. A good reminder of what can be done together as family at home.

Relaxed and natural setting
The key aim was to showcase what students could do, but we wanted to demonstrate what students are doing in lessons every day. Our teachers utilised their everyday resources and tools and didn't need to create new materials to use and display. Creating a relaxed setting provided an environment where parents and teachers were able to interact and discuss maths, we hope this contributed to a shift in their outlook and approach in their thinking towards maths.

Timing and consistency
It was important to make the night accessible, so we opened classrooms from 5pm – 6.45pm. We encouraged families to visit all of the classrooms so they could see the sequence and progression of ideas from Prep to Year 6.

Enjoyable
The use of raffles and lucky door prizes added a little atmosphere to the evening.

FEEDBACK

We had over 100 families attend. Families were actively engaged, with many discussing how they could easily undertake these activities at home. It was a great opportunity for parents and teachers to interact in an informal setting. By using the games that students had experienced, we achieved our goal of having students lead the activities.

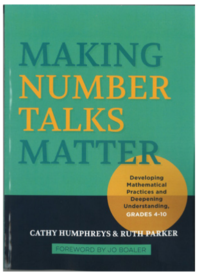
The role of a school is ever evolving, we educate our students, but also educate families and it was great to share the understanding that it is ok to make mistakes when solving a problem. Family Maths Nights are the perfect opportunity to provide parents with a reminder of what they can do to with their children at home and allow them to have a positive approach and outlook on maths.

If you are keen to run a Family Maths Night at your school, MAV has developed a fantastic resource that contains many games, instructions and templates, the *Family Maths Night - School Support Resource* is available from the MAVshop.

Visit www.mav.vic.edu.au/shop.

MAKING NUMBER TALKS MATTER

Genovieve Grouios - Mathematics consultant



Communication in the mathematics classroom plays a significant role in students learning and development.

This is often encouraged through class or group discussions, where students have the opportunities to engage in creative and meaningful mathematical experiences.

However, whilst this may sound promising, many teachers would agree that discussions can often be a difficult task to manage. By this I mean, it is difficult to ensure that discussions are always purposeful and lead towards intended goals. For instance, we cannot guarantee that all students will participate in conversations due to time constraints and their self-confidence. In this situation, it is common for students with high mathematical abilities to confidently participate, whilst others who lack this confidence have the tendency to simply tune out or request if they can be excused from the classroom to avoid these fearful experiences. What this demonstrates is that, we cannot simply ask students for an answer and move on or tell students how to solve a problem. Rather, students need to learn how to make sense of situations they come across and engage in discussions that delve deeper into the mathematical journeys that they take. Here the focus is on how students think logically or mathematically about different situations.

During a number talk, students are asked to think about ways to solve various computation problems mentally. What many teachers will find is that students frequently resort to visualising a written strategy or formula. For example, a common response for the question below is:

$$\begin{array}{r} 30 \\ -18 \end{array}$$

You cannot subtract 8 from 0 (when in fact you can) so you take/borrow a 1 from the 3 and make this 2. You put the 1 next to the 0 to make 10. Now we can subtract 8 from 10 to get 2. Next we take 1 from 2 to get 1. The answer is 12.

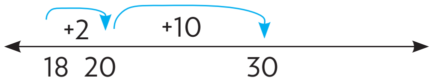
Notice that the student in this case, relies heavily on memorising procedures with little understanding about place value, numbers and how operations work. During a number talk, you can look beyond this method and focus more intently on basic facts, mathematical language and student invented strategies.

Making Number Talks Matter is a great text which explores creative ways to encourage all students to talk about their understandings, justify their responses and make sense of mathematical concepts they encounter. The text shows teachers how to effectively plan for a productive class discussion and develop guiding questions to prompt student thinking.

Time or speed mathematics is not a focus, which portrays a powerful message to students and helps them change their perceptions on what it means to be good at mathematics. What this text offers, are ideas to help each individual teacher conduct a fifteen minute (give or take) number talk that is specifically tailored for their students.

Humphreys and Parker suggest some guiding principles to successfully conduct a number talk.

- Each number talk **does not require any materials**. That is, all paper and pencils are set aside so that students learn to veer away from rote procedures.
- Teachers are encouraged to **write equations or expressions horizontally** on the board to discourage students from using pen and paper procedures i.e. 30 - 18 =
- **Allow ample time** for students to think deeply about the problem. Instead of raising hands, **students make a fist on their chest** and put up their thumbs when they have a solution.
- When most thumbs are up, start asking for answers and **record all answers** (even those which are incorrect). These representations of students thinking should be presented to the whole class. You can use visuals or mathematical notation to help demonstrate how the student was thinking about the problem. In the example (left), number lines or a hundreds charts would help students articulate their findings e.g. a response from a students could be *I added 2 to 18 to make 20 and added 10 more to make 30. Altogether I added 12*. Here the student used the 'think addition' strategy.



- Once students have given their answers, prompt them to **explain how they obtained them** i.e. students describe the method they used and explain why this makes sense. In this section, students should not look to the teacher for answers instead they should be communicating with their peers. Hence, the teacher's role is to ask guiding questions to help conversations flow e.g. does anyone have a question for...? who would like to add to...idea?
- After a student shares a strategy, **use open-ended or probing questions to help them articulate their ideas clearly** and defend their strategy. This is also a good opportunity to explore and use key mathematical vocabulary and correct any errors in language
- Display these posters so that the class can refer to them in future lessons.

Some of the key goals of the Victorian Curriculum: Mathematics are to:

a) help students become lifelong learners with sophisticated and refined mathematical capabilities and

b) provide students with the skills and knowledge to make informed decisions, respond effectively in unfamiliar situations and solve problems efficiently.

This book is a great resource to help students develop into numerate citizens, where they learn to build their self-efficacy and confidence and transfer their skills and knowledge of mathematics across situations outside the mathematics classroom. It is suited for both primary and secondary levels and can be used by teachers to help integrate the proficiency strands in daily lessons.

Making Number Talks Matter is available from the MAVshop.

CLASSPAD AND SPECIALIST MATHS

Maria Schaffner, Tom Murphy and Theo Vlantis - Mathematics teachers, Penleigh and Essendon Grammar School

EFFECTIVE USE OF CAS FOR OPTIMUM EXAM RESULTS

In Specialist Mathematics students have to be very proficient in solving problems using pen and paper. This is not only because one of the VCE exams does not allow any form of technology, but in this way students can actually get through the required steps and understand the method(s) that is/are used in obtaining the required answer. However, it is crucial to be able to distinguish when to use technology (when it is allowed). This will not only allow students to get quicker to the answer (with fewer or no errors), but to save valuable time in order to finish the exam or even have time to check the steps to some of the more complicated algebraic questions. This should not eliminate the understanding of the concepts involved in solving the problem but to get rid of tedious and lengthy calculations.

The following multiple choice examples show that by using CAS the problems are solved faster, with more accuracy and without taking away the student's understanding of the question and the concepts involved in obtaining the answer. This helps in quickly eliminating some options and then using CAS to finish solving the question. However, as it is shown below there can be the odd multiple choice that can be answered without any theory knowledge but only with the understanding of calculator use.

PARTIAL FRACTIONS

2019, VCAA Exam 2, Question 2

The asymptote(s) of the graph of $f(x)=\dfrac{x^2+1}{2x-8}$ has equation(s)

A. $x=4$ B. $x=4$ and $y=\dfrac{x}{2}$

C. $x=4$ and $y=\dfrac{x}{2}+2$

D. $x=8$ and $y=\dfrac{x}{2}$ E. $x=8$ and $y=2x+2$

Equating the denominator to zero will reveal that the vertical asymptote is where $x=4$ and quickly eliminate options D and E.

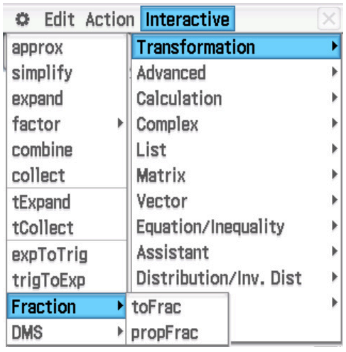
While $f(x)$ can be split by using tedious long division to get it into the form

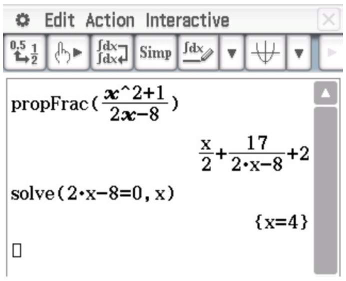
$f(x)=\dfrac{x}{2}+2+\dfrac{17}{2x-8}$

(giving option C as the answer)

Using CAS:

Type and highlight the expression $\dfrac{x^2+1}{2x-8}$ → Interactive → Transformation → Fraction → propFrac we obtain:





USE OF SUMMATION SYMBOL Σ

2019, VCAA Exam 2, Question 4

The expression $i^{1!}+i^{2!}+i^{3!}=\ldots+i^{100!}$ is equal to

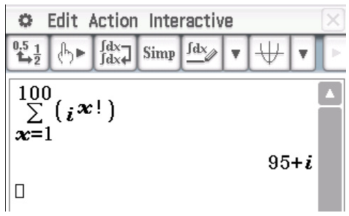
A. 0 B. 96 C. $95+i$ D. $94+2i$
E. $98+2i$

While there is an algebraic way, it would take some thinking and time to obtain the answer. However, this particular question, which happens to appear on rare occasions, can be answered using CAS with limited understanding of the skills.

Using CAS:

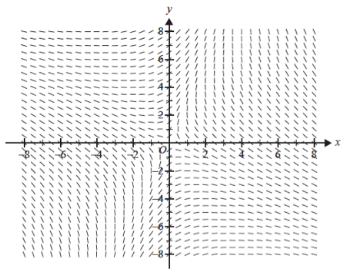
The summation template will give the answer quickly, but you must ensure that the complex mode is set to avoid the non-real error.

Soft Keyboard → Math2 → → enter the information as shown to obtain answer C.



DIFFERENTIAL EQUATIONS: SLOPE FIELDS

2018, VCAA Exam 2, Question 10



The differential equation that best represents the direction field above is

A. $\dfrac{dy}{dx}=\dfrac{2x+y}{y-2x}$ B. $\dfrac{dy}{dx}=\dfrac{x+2y}{2x-y}$

C. $\dfrac{dy}{dx}=\dfrac{2x-y}{x+2y}$ D. $\dfrac{dy}{dx}=\dfrac{x-2y}{y-2x}$

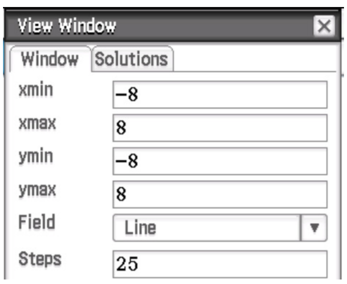
E. $\dfrac{dy}{dx}=\dfrac{2x+y}{2y-x}$

By observing that there are vertical gradients along $y=2x$, options C and E are eliminated. This leaves options A, B and D as possible answers.

The Casio Application that sketches slope fields will help identify the correct answer here. This is another example where technology can be used to answer the question without any algebraic steps.

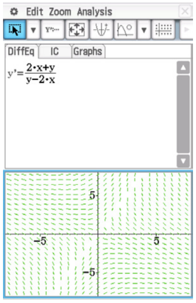
Using CAS: Menu → DiffEq-Graph Enter diff eq → graph

The slope field will appear. Comparing the three graphs gives the answer A. Ensure that the window setting is adjusted as shown.

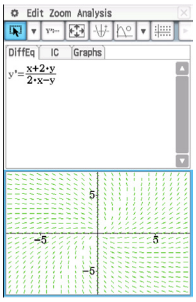


Note: Increasing the number of steps will increase the density of the slope field.

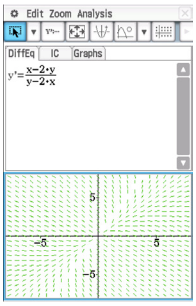
Option A



Option B



Option D



STATISTICS

2019, VCAA Exam 2, Question 18

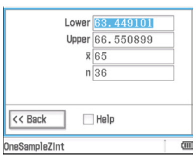
The masses of a random sample of 36 track athletes have a mean of 65 kg. The standard deviation of the masses of all track athletes is known to be 4 kg.

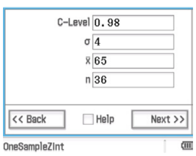
A 98% confidence interval for the mean of the masses of all track athletes, correct to one decimal place, would be closest to:

A. (51.0, 79.0) B. (63.6, 66.4)
C. (63.3, 66.7) D. (63.4, 66.6)
E. (64.3, 65.7)

Students can choose to use the formula supplied by VCAA, insert the appropriate numbers and obtain the answer (C).

However, using CAS time can be saved as: CAS: Menu → Statistics → Calc → Interval → choose variable, next and then insert numbers as required to obtain:





COMPLEX NUMBERS

2017, VCAA Exam 2, Question 5

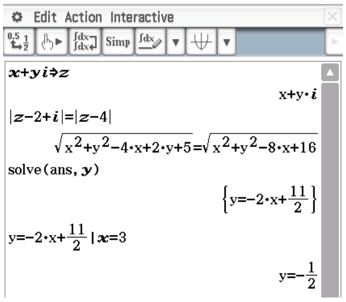
On an Argand diagram, a point that lies on the path defined by $|z-2+i|=|z-4|$ is

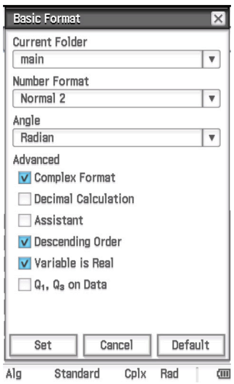
A. $\left(3,-\frac{1}{2}\right)$ B. $\left(-3,-\frac{1}{2}\right)$ C. $\left(-3,\frac{3}{2}\right)$
D. $\left(3,\frac{1}{2}\right)$ E. $\left(3,-\frac{3}{2}\right)$

This would take some time substituting $z=x+yi$ and then algebraically reducing to the equation $y=-2x+\dfrac{11}{2}$, but

Using CAS: Define $x+yi \Rightarrow z \rightarrow |z-2+i|=|z-4|$ EXE → Solve(ans, y) → then insert x values to check y values to obtain (answer: A)

Note: Ensure that the Variable is Real has been ticked in the Basic Format settings





CALCULUS

2017, VCAA Exam 2, Question 6

Given that $\dfrac{dy}{dx}=e^{x}\arctan(y)$, the value of $\dfrac{d^2y}{dx^2}$ at the point (0, 1) is:

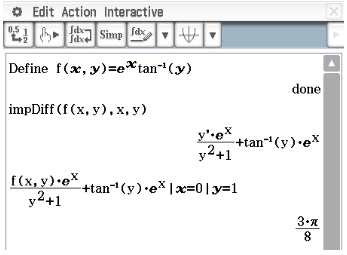
A. $\frac{1}{2}$ B. $\frac{3\pi}{8}$ C. $-\frac{1}{2}$ D. $\frac{\pi}{4}$ E. $-\frac{\pi}{8}$

This needs to be differentiated implicitly and as the expressions are somewhat complicated, algebraic errors can occur.

Using CAS:

Define $f(x, y)=e^{x}\tan^{-1}(y)$ → interactive → calculation → *impDiff(f(x, y), x, y)*

Then substitute $f(x, y)$ for y' and $x=0, y=1$ and execute to obtain the answer B.



It is therefore important as teachers that throughout the VCE years students are directed to use technology to its full potential. Practising the steps for particular procedures must become a regular routine and should also form part of the bound book. The better the students know how to use CAS for the specific situations the more effective CAS will be to optimise exam results.

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VCAA. (2019). *Specialist Mathematics Written examination 2*. Melbourne: Victorian Curriculum and Assessment Authority.

VCAA. (2018). *Specialist Mathematics Written examination 2*. Melbourne: Victorian Curriculum and Assessment Authority.

VCAA. (2017). *Specialist Mathematics Written examination 2*. Melbourne: Victorian Curriculum and Assessment Authority.

TI-NSPIRE AND SPECIALIST MATHS

Stephen Crouch - Frankston High School and James Mott - Suzanne Cory High School

THE ROLE OF TI-NSPIRE IN SPECIALIST MATHS

Mathematics is a discipline that not only requires astute knowledge of algebraic and formulaic content, but also a deep visual and dynamic understanding of how various concepts work together to produce results. The TI-Nspire CX CAS Handheld and Computer Software suite are very well suited to enable students and teachers to explore in class, at great depth, complicated mathematical ideas; as well as efficiently find solutions to problems during time-sensitive assessments such as end-of-year exams. Wise use of this technology can greatly enhance learning and maximise exam results.

PARTICLES COLLIDE: A GRAPHICAL APPROACH

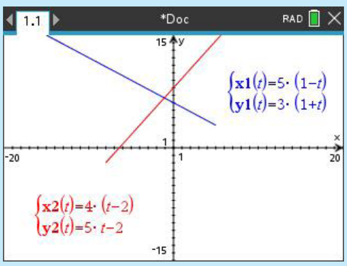
As part of the Vector Calculus component of the syllabus, students are expected to perform operations with the position vector of an object, as a function of time, and be able to sketch the corresponding path of the object where it is defined parametrically. A useful feature of the TI-NSpire CX CAS series is the ability to sketch the path of an object when its position is defined parametrically.

This will be demonstrated using question 4 from the 2016 Specialist Mathematics Exam 2 paper.

2016 Exam 2 Question 4
Two ships, A and B, are observed from a lighthouse at origin O.
Relative to O, their position vectors at time t hours after midday are given by
$\underset{\sim}{r}_A = 5(1-t)\underset{\sim}{i} + 3(1+t)\underset{\sim}{j}, \qquad \underset{\sim}{r}_B = 4(t-2)\underset{\sim}{i} + (5t-2)\underset{\sim}{j}$
where displacements are measured in kilometres.

b. Sketch and label the path of each ship on the axes below.
Show the direction of motion of each ship with an arrow. 3 marks

Upon opening a Graphs Page, press **Menu > Graph Entry/Edit > Parametric**. Once here you are able to enter in your parametric equations.

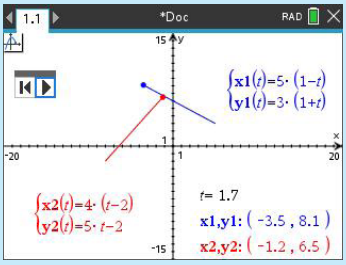


When examining the motion of two objects we are often interested in three things:

1. What is the direction of motion of each particle?
2. Do the objects paths cross? If so when and where?
3. Will the objects collide? If so when and where?

It can be difficult for students to understand the subtle difference between points 2 and 3, especially when exploring such a problem in a static environment.

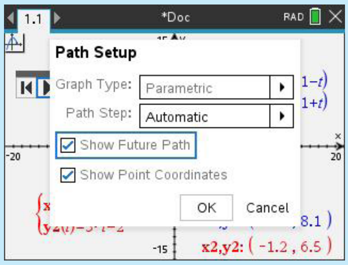
A new and powerful feature of the TI-Nspire CX CAS II is the ability to observe the motion of an object along its (parametrically defined) path. To observe both particles move, after entering in your parametric equations press **Menu > Trace > Path Plot > Parametric**. A box will appear with a 'play' button and a 'reset' button. Move your cursor over to the play button and click on it to begin the animation.

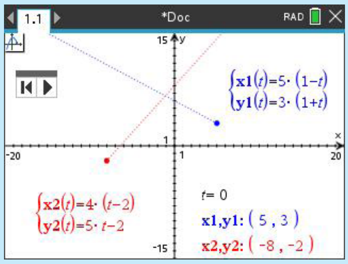


As the particles move, the CX CAS II displays the value of the parameter along with the corresponding coordinates of each particle's position. The visual representation affords students of all abilities an additional entry point to engage with the problem, and the dynamic representation can help students understand the subtle difference between 'paths crossing' and 'particles colliding'.

By using the TI-Nspire CAS CAS II to sketch the parametric curves and using the **Path Plot** feature, students of all abilities are able to answer exactly what the question is asking of them in an efficient way, and without having to perform any calculations or algebraic operations.

The **Path Plot** feature not only shows the direction of motion of a particle, but also generates the particles path. The CX CAS II also has the ability to show the path of the particle prior to commencing the **Path Plot** animation. To see this press **Menu > Trace > Path Plot > Path Setup**. You will need to change the graph type to Parametric, and tick the 'Show Future Path' box. Once the settings are updated and you go to observe the motion of the particles - **Menu > Trace > Path Plot > Parametric** - you will now see the paths that the particles will eventually move along, in addition to all the other aforementioned information.





DIFFERENTIAL EQUATIONS USING TI-NSPIRE

Differential Equations is one of the core ideas that is explored in Specialist Maths. This topic is central to most mathematical analysis, whether it be population growth, pendulum motion or the deflection in beams. In the Differential Equations section, students are expected to solve both first-order and second-order differential equations, both as a function of the dependent and independent variable, as well as formulate differential equations from various contexts, especially regarding rates of change (e.g. the scenario of substances such as salt being mixed with a liquid).

The TI-Nspire CX CAS includes a useful command '**deSolve**' which can be used to efficiently and accurately produce solutions of differential equations. This command produces both general and specific solutions, depending on the information provided in the question.

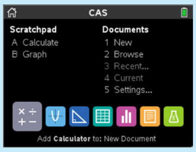
First-order differential equations

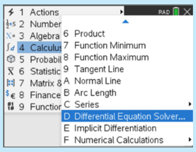
The use of **deSolve** for first-order differential equations can be shown using the scenario explored in Question 3 from the 2017 Specialist Mathematics Exam 2 (NHT):

Bacteria are spreading over a Petri dish at a rate modelled by the differential equation $\frac{dP}{dt}=\frac{P}{2}(1-P), 0<P<1$, where P is the proportion of the dish covered after t hours.

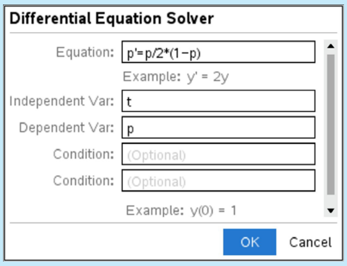
Both a general and specific solution to this differential equation can be found, with part **a. iii.** of the question specifying the initial condition – that half of the Petri dish is covered by the bacteria at $t=0$.

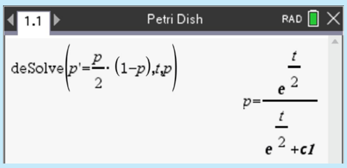
Within a Calculator page, press **Menu > Calculus > Differential Equation Solver**..., as shown below.



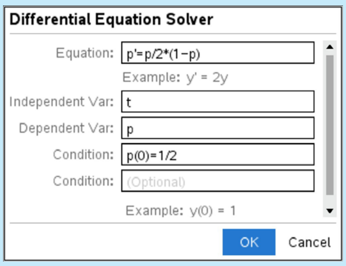


This opens a dialog box, in which you can enter the differential equation, its variables, and if given, initial condition(s). Entering these in for our scenario, and pressing **Enter** after choosing OK:





The GUI nature of the Differential Equation solver dialog box allows students to quickly enter information without worrying about syntax. This is a feature introduced in the CX-II CAS. Using the initial condition produces the specific solution, which can be used to quickly verify the work done 'by-hand' in the exam:

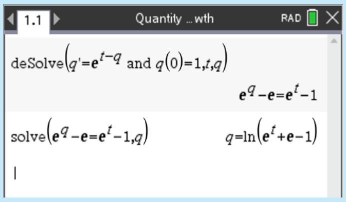


The use of **deSolve** for first-order differential equations can be also shown using the scenario explored in Question 3 from the 2019 Specialist Mathematics Exam 2:

The growth of a quantity Q with respect to time t is modelled by the differential equation $\frac{dQ}{dt}=e^{t-Q}$ where $t \geq 0$ and $Q=1$ when $t=0$.

The specific solution can be found, with part **b. ii.** requesting to show that $Q=\log_e\left(e^t+e-1\right)$.

Within a **Calculator page, press Menu > Calculus > Differential Equation Solver**..., then enter the appropriate information into the dialog box. The resulting equation is not in the required form, however using the **solve** command (**Menu > Algebra > Solve**) takes care of the final step:



This result from the TI-Nspire verifies the result requested in the exam.

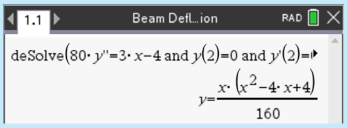
Second-order differential equations

The use of **deSolve** for second-order differential equations can be shown using the scenario explored in Question 5 from the 2018 Specialist Mathematics Exam 2 (NHT):

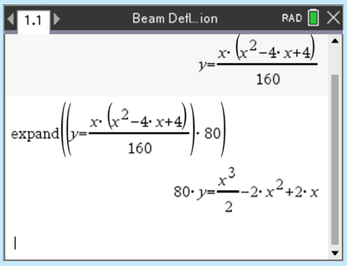
A horizontal beam is supported at its endpoints, which are 2m apart. The deflection y metres of the beam measured downwards at a distance x metres from the support at origin O is given by the differential equation $80\frac{d^2y}{dx^2}=3x-4$.

The specific solution to this differential equation can be found, with part **a.** giving the information $\frac{dy}{dx}=0$ and $y=0$ at $x=2$, and requesting to show that $80y=\frac{1}{2}x^3-2x^2+2x$.

As before, within a Calculator page, press **Menu > Calculus > Differential Equation Solver**..., then enter the appropriate information:



However, given that the question requires $80y$ to be the subject, careful use of the **ans (Ctrl > (–))** and **expand (Menu > Algebra > Expand)** commands leads to the desired result:



The resulting functions can therefore be analysed and a graph drawn/major points found, and so on.

CONCLUSION

The TI-Nspire CX CAS offers a range of tools to users of all abilities to engage with examination-style questions by alleviating the algebraic processes and allowing the user to focus on interpreting and understanding the underlying concept(s) of a question. With added dynamic functionality in the graphs page, the TI-Nspire CX CAS-II can provide greater access to learners and evoke rich classroom discussion. The dynamic nature of the aforementioned features of the TI-Nspire CX CAS provides all users a great tool to be teaching, learning, and doing mathematics in the 21st century.

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VCAA. (2016). *Specialist Mathematics.*
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MATRICES

Andrew Stewart

SACs are a great opportunity to set more difficult problems than are encountered in exams of any sort, due to greater time being allocated to the solution of the task.

Transition matrices of various sizes have been featured in trial examinations for many years. However, recent additions to the topic such as dominance, permutation and communication matrices can add challenging variety to a SAC Task.

DOMINANCE MATRICES

Building a task based on a Trial Examination question data.

2016, Examination 2, Question 3
Four of the permanent staff played a round-robin darts tournament during their lunch breaks, in which every staff member played against every other staff member once. In each game there was a winner and a loser.

A table of one-step and two-step dominances was prepared to summarise the results.

Employee	one-step dominances	two-step dominances
Ken (*K*)	2	2
Leanne (*L*)	1	1
Maggie (*M*)	2	3
Neil (*N*)	1	2

Suggested activities

(a) How many games did .... win ? (Choose a team, an easy starter)

(b) What is the final ranking of these teams?

(c) From considering the one-step and two-step dominances, explain how you know that
i. Leanne defeated Neil
ii. Neil defeated Maggie
iii. Maggie defeated Ken

(d) The information from part (c) is included in the partly completed one-step dominance matrix shown.

$$\begin{array}{cc} & \textit{loser} \\ & \begin{array}{cccc} K & L & M & N \end{array} \\ \textit{winner} \begin{array}{c} K \\ L \\ M \\ N \end{array} & \begin{bmatrix} \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & 1 \\ 1 & \ldots & \ldots & \ldots \\ \ldots & \ldots & 1 & \ldots \end{bmatrix} \end{array}$$

Complete the matrix.

(e) Show the game results for this competition in a directed graph in which the arrow points from the winner towards the loser.

(f) (Alternative to (b))

Construct the sum matrix (sum of one-step and two-step dominance matrices) and hence determine the winner of this competition.

Even if students cannot complete part (c), they should not be disadvantaged for parts (d) and (e). This kind of task becomes more challenging if dealing with a five or six team competition.

Rather than take the final results and work back, start in the middle ...

2017, Modified MAV Trial Examination 2, Question 2
The staff at Scotty Moffit Motors have a darts competition every Friday afternoon between teams led by Alana (*A*), Barry (*B*), Chris (*C*) and Dana (*D*). The results after two rounds have been played (where every game has a winner and a loser) are shown in the one-step dominance matrix below.

$$\begin{array}{cc} & \textit{loser} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ \textit{winner} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(a) Which teams will play each other in the third round, so that each team has played each of the others once and once only.

If Chris' team wins their third-round game, they have a chance to win the competition being the best team as determined by the sum of the one-step and two-step dominances. However, if a particular team wins the other third round game, Chris' team will finish second.

(b) Write down the leader of the team that by winning their third-round game will finish first ahead of Chris' team.

(c) Write down the final dominances table for this situation.

Other suggested activities

(a) What does this matrix tell us about the results of the first two rounds?
(Use before (a) above)

(b) Construct the one-step, two-step and sum dominance matrices for
i. Chris' team winning, and winning the competition
ii. Chris' team winning, but coming second
iii. Show the game results and all dominance matrices for the worst possible overall result for Chris' team.

PERMUTATION MATRICES

A key resource for permutation matrix problems is a source of anagrams. I use a book to find combinations of four, five or six letters which can be re-arranged to form the greatest number of different words. There are many anagram websites available.

Usually permutation problems are in Exam 1 (multiple choice), but they can be adapted to form a SAC task. A good starting point is to determine the word formed by a simple multiplication using five or six letters to make students work a little harder.

2016, Modified MAV Trial Examination 1, Question 3

If matrix $K = \begin{bmatrix} E \\ D \\ I \\ T \end{bmatrix}$

and matrix $P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

then what word is formed by the matrix product *PK*?

Explanation questions can be quite challenging.

2017, Modified MAV Trial Examination 1, Question 4

If matrix $K = \begin{bmatrix} O \\ P \\ T \\ S \end{bmatrix}$

and matrix $P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

explain why the matrix $\begin{bmatrix} O \\ P \\ T \\ S \end{bmatrix}$ is formed by the product P^2K.

If a permutation matrix is raised to a power which is the same as the number of letters which change position each time, it becomes the identity matrix and the product is identical to the starting matrix. In this case only two letters move each time.

A final challenging part would be to construct the whole permutation matrix that when multiplying a particular set of letters gives another particular set of the same letters. This could be made more challenging by raising the permutation matrix to a power.

2017, Modified MAV Trial Examination 1, Question 4

If matrix $K = \begin{bmatrix} O \\ P \\ T \\ S \end{bmatrix}$

and matrix $P = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$,

fill in the element values for matrix P so that the matrix product P^3K gives the matrix $\begin{bmatrix} P \\ O \\ S \\ T \end{bmatrix}$.

COMMUNICATION MATRICES

These kinds of problems can be quite tricky to write, and hence have not appeared often in trial examinations. When writing tasks, I often take an example from an examination or textbook and modify it to suit my purpose.

In the multiple-choice situation, either a matrix is provided and students are required to identify the correct diagram for that matrix, or the diagram is provided and students are required to identify the correct matrix.

The following example from a MAV Trial paper has been modified to show both ways. In a SAC, a larger matrix would provide a greater challenge.

2018, Modified MAV Trial Examination 1, Question 6

Kerri (K), Leanne (L), Maree (M) and Noni (N) are four key managers in a business who should be in communication with each other all the time. Unfortunately, personality clashes have meant that some of these managers will not directly communicate with some of their colleagues – either sending or receiving messages from their colleague. The communication matrix represents this current situation.

$$\begin{array}{cc} & \text{to} \\ & \begin{array}{cccc} K & L & M & N \end{array} \\ \text{from} \begin{array}{c} K \\ L \\ M \\ N \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

The elements in the matrix are such that

- 1 indicates that direct communication from one person to another is possible
- 0 indicates that direct communication is not possible.

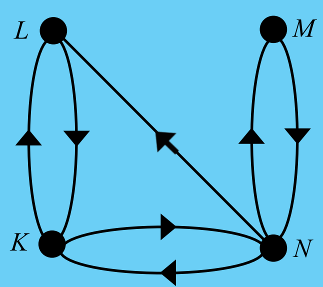
Draw a diagram to represent this situation, with the arrow pointing in the direction of the communication.

2018, Modified MAV Trial Examination 1, Question 6

Kerri (K), Leanne (L), Maree (M) and Noni (N) are four key managers in a business who should be in communication with each other all the time. Unfortunately, personality clashes have meant that some of these managers will not directly communicate with some of their colleagues – either sending or receiving messages from their colleague. The diagram alongside represents this current situation.

Construct a communication matrix to represent this situation. The elements in the matrix should be such that

- 1 indicates that direct communication from one person to another is possible
- 0 indicates that direct communication is not possible.



These problems can be extended by asking students to find the chain of communicators so that two individuals (who cannot directly communicate) can pass a message to one another. This is best done where the matrix is at least 5 × 5 or bigger.

Can you set up a large (5 × 5 or bigger) matrix in which two individuals cannot communicate with each other using any/all other members?

FLEXIBLE LEARNING SPACES

Raelene Harding - Year 1 teacher, Holy Spirit School, Lavington

With the rise of the 21st century, educational landscapes have changed. There has been a shift from the traditional, four wall classrooms with a closed door, to a more open environment, providing students with flexible physical spaces that inspire learning.

With this environmental shift changes in teacher pedagogy from delivering content to focusing on student learning for deep, transferable knowledge has also been brewing. This article will focus specifically on suggestions for changing the educational space to maximise student learning of mathematics. It is important for teachers of mathematics to create meaningful experiences for their students to engage in mathematical learning.

Creating an effective environment for mathematical learning in a flexible learning space calls for innovation. Strom (2019) advocates that, 'quality instruction builds students' conceptual knowledge through active learning, authentic problem solving and student – led solutions (Strom 2019). Therefore, there is a need to design learning spaces that allow for learning in this manner. In addition, learning spaces that foster student engagement and provides a sense of belonging helps to build a learning community connecting students and the teacher/s together.

My learning space is organised into three focus spaces to reflect the above statements.

CENTRAL SPACE

The first space is known as the central space and is a large open area on the floor.

This central area is for whole group vignettes and to put a spotlight on important mathematical concepts for learning. This space is a safe space for students to share their thinking, a place to reason, to challenge and to share their strategies with the whole group.

The central space permits the practice of direct instruction and the launch phase of Peter Sullivan's (2018) lesson structure.

INVESTIGATE, EXPLORE, DISCOVER SPACE

The second space in our area is known as investigate, explore and discover. This space is for investigation and exploration, leading to discovery. In this space students work collaboratively like mathematicians, as they investigate and explore mathematical concepts and ideas.

In this space, students have free access to an array of manipulatives to assist them. This space empowers students to work together, to test their thinking and conjectures and to prove to each other their discoveries and understandings.

My role when students are working with their peers in this area is to facilitate and conference with the students to ascertain their current level and to support higher order thinking. The investigate, explore and discover space along with the quiet space engages students in authentic problem solving and challenging tasks that are open tasks allowing for multiple entry points and solution paths.

Sullivan's (2018) explore and summary phases are entered during this learning space.

Students have access to desks, chairs, soft furnishings as well as the floor. All manipulatives are placed on open shelving at the student's height for the them to access and use as they please.

Students have access to a wide variety of resources to support their exploration, investigation and discoveries. Such as

- blocks
- variety of counters
- calculators
- paper and writing materials
- number charts, number lines, bead strings
- rulers, tape measures, string
- dice of all sorts
- data jars
- 2D and 3D shapes, tangrams, everyday materials such as boxes, containers
- clocks, timers, sand timers, stop watches
- coins and shop tags.

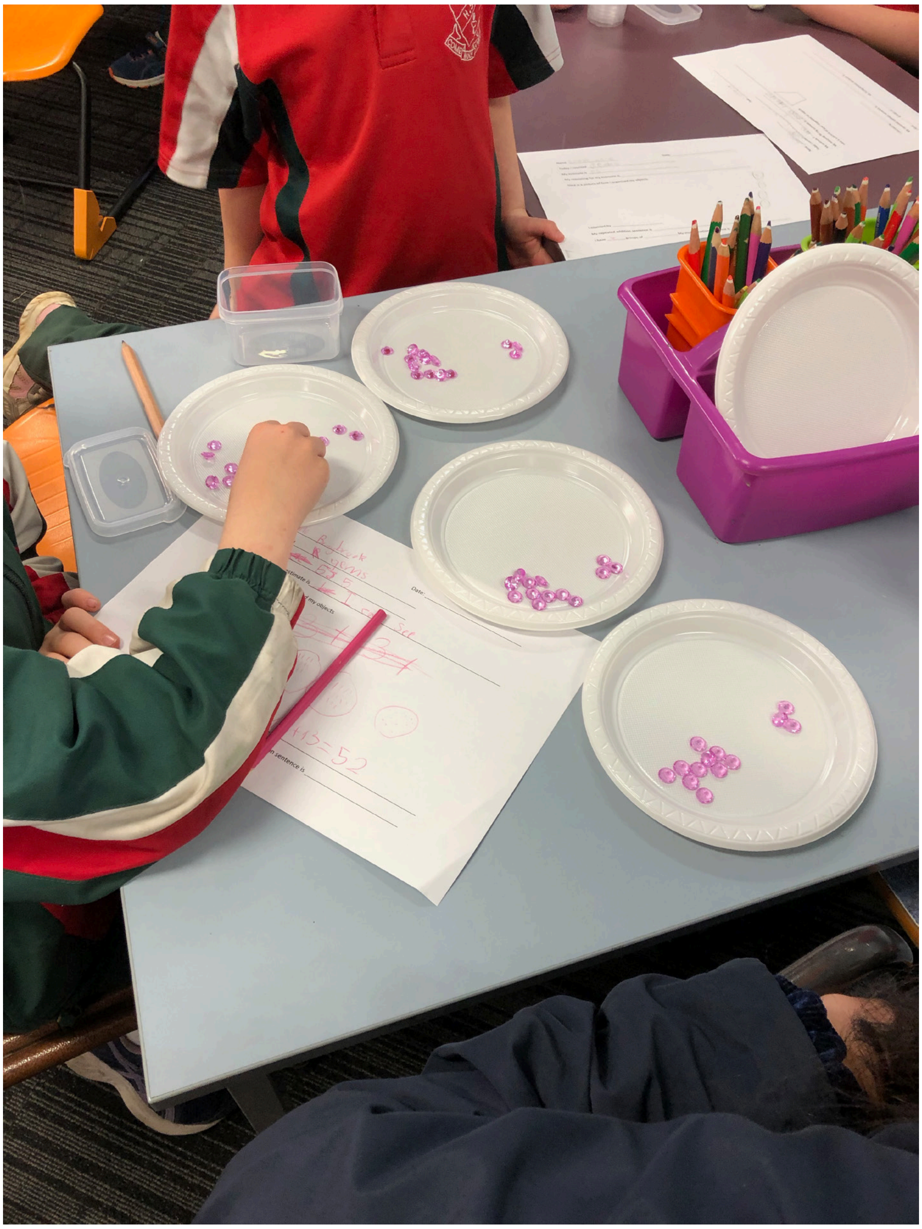
QUIET SPACE

The third space in our learning environment is a quiet space for students who wish to work on their own. If students chose to use this space, they have access to all materials. I have found students like to enter this space when they are grappling with a concept and need some time to think on their own.

When students enter this space, I see this as an opportunity to conference with my students to see how they are going and if I can assist in anyway. The students are free to move the furniture to create a nook where they can work quietly. Often students will move the soft furnishings such as ottomans to create a cave or work under desks and move chairs to close a space around them.

Setting up these spaces with my students gives them a greater sense of ownership over their learning. The environment is fluid and students can enter and leave when they feel the need. The only compulsory learning space is the first central area space.

These spaces are adaptable and can be implemented at any year level.



I believe organising the learning space in this way, fosters student engagement and provides a sense of belonging. It also allows access for a variety of pedagogical practices to be implemented.

The restructuring of my physical learning space to accommodate for the three spaces I have created in my classroom has reshaped my teaching and instructional practice. Post assessment results show that students have made immense growth. I feel I have created a learning space where the class can present, meet, discuss, challenge and listen to each other to enhance their learning. I believe integrating best practice and educational research is a recipe for learning.
Learning = teaching + time.

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INVENTION OF SYMBOLIC ALGEBRA

Robyn Arianrhod - Affiliate, School of Mathematics, Monash University

Remember the old childhood game where someone asks you to pick a number, and then leads you through a series of arithmetic instructions and ends up guessing the answer you've come up with? Perhaps it still lives on in playgrounds or junior classrooms, but the game I'm thinking of goes something like this:

> 'Pick a number, any number. Don't tell me what it is, but now add six to it. OK, now double this new number. Got it? Good, now take away eight. And now take away twice the number you first thought of. No, don't tell me... I bet the answer is four!'

It seems like magic when you're a kid, but of course the game has been algorithmically programmed from the beginning. To my mind, though, the real magic happens when you write the algorithm as a symbolic algebraic equation:

$$2(x+6)-8-2x=4.$$

For a start, it strips away the conjuror's mask by showing how easy it is to make up such puzzles – any similar equation will do. But the really amazing thing is that you can see the whole set of operations at a single glance. There's no need to hold all the instructions in your head.

Of course, this is a pretty simple puzzle, but you can see the point. Even learning multiplication tables is so much simpler if we can see the pattern visually: compare 'one times three is three, two times three is six, three times three is nine...' with the readily apparent patterns we're now used to:

$$1 \times 3 = 3$$

$$2 \times 3 = 6$$

$$3 \times 3 = 9$$

and so on.

I say 'the patterns we're used to,' but the ability to see patterns easily by using universally accepted symbols is a relatively late arrival in the history of mathematics.

For a start, today's universally used numerals – which evolved from earlier Hindu-Arabic systems – didn't obtain their modern form until the sixteenth century, and it was only then that they fully replaced Roman numerals in Europe.

Before then, there were no common signs for the basic operations of arithmetic, either. The $+$ sign first appeared in print in 1489, in Germany, although it did not quite mean the operation of addition; the first to use it as such seems to have been Michael Stifel in 1544, but even in 1600 most people wrote out 'plus' and 'minus' – or sometimes 'p' and 'm'. They also wrote 'equals' and 'times'. Readers of Terence Mills' recent article in *Common Denominator* will recall that the $=$ symbol was created by Robert Recorde in 1557, but that it took decades to catch on. The symbol $\times$ was first used, by William Oughtred, only in 1631.

As for algebra, René Descartes was the first to publish the use of x for the unknown number, in 1637. Before then, most people had written and solved equations as word problems, like the game I mentioned earlier, and instead of x they would write 'the unknown number' or something similar.

So you can see that it took a long time before it was possible to write even such a basic algebraic equation such as $2(x+6)-8-2x=4$.

THOMAS HARRIOT

A few mathematicians over the centuries had made a start on trying to simplify things by using letters or other symbols for unknowns – especially François Viète in the late 1500s. But the first to write recognisably modern algebraic equations was Viète's disciple Thomas Harriot. Harriot didn't publish his many discoveries in mathematics, physics, and phonetics, but his friends published a posthumous partial account of his algebra in 1631. It was the first ever fully symbolic algebra text.

Harriot's manuscripts show that by 1600, he was routinely using lower case letters from the beginning of the alphabet for the unknowns. He sometimes used n, too, especially for integers. He was an early adopter of Recorde's $=$ symbol (although like Recorde's it was longer than our modern version), and he created the prototype of the symbols for 'less than' and 'greater than'; in fact, the $<$ and $>$ signs first appeared in print in Harriot's posthumous book.

He used a dot for multiplication or else he just ran numbers or symbols together, the way we do today when we write $2x$ instead of $2 \times x$. Similarly, he represented powers by repetitions, rather than the index notation later introduced by Descartes. Which shows that not all symbols are equally good: writing a^6 is neater than writing $aaaaaa$.

Perhaps most important of all, Harriot created the method of writing all the terms in a polynomial equation on the left-hand-side with zero on the right: for example, $4n^2+4n+1=0$ (which he wrote as $4nn+4n+1=0$). Actually, having a symbol for zero – and treating it like a number rather than an absence, a 'nothing' – was a remarkable achievement in itself, usually credited to Brahmagupta in 621 CE.

As today's students know, finding solutions by factorising is more readily apparent for $4n^2+4n+1=0$ than it is for equivalent forms such as $4n^2=-4n-1$. Harriot also realised that if the solutions of a cubic equation are l, m, p, then the equation can be factorised as $(n-l)(n-m)(n-p)=0$, and so on for higher powers. The gist of the 'factor theorem' seems self-evident in hindsight, but history shows that for thousands of years, many seemingly simple things were far from obvious.

As another simple example, take the binomial theorem. It seems to have taken 1000 years for mathematicians using verbal algorithms to pass from knowing the expansion of $(a+b)^2$ to that of $(a+b)^3$, and another 1000 years before anyone wrote down the symbolic *general* form of the coefficients in the expansion of $(a+b)^n$ – namely,

$$1, n, \frac{(n)(n-1)}{2}, \frac{(n)(n-1)(n-2)}{3\times2\times1}, \frac{(n)(n-1)(n-2)(n-3)}{4\times3\times2\times1},$$

and so on. This 'anyone' was Harriot – again!

Four hundred years on, Maths Methods students routinely use these expressions in studying combinations and probability, as well as in binomial expansions.

As for Harriot, he also realised that the general form of the binomial coefficients suggest there's no reason to stick with positive integer values of n, as you have to do if you rely on Pascal's triangle (which, incidentally, was known in mediaeval China and the Middle East, and then Europe, long before Blaise Pascal's fuller development in the seventeenth century).

In other words, while knowledge of binomial coefficients had evolved in concrete, whole number contexts – such as finding the number of ways of choosing three cards from a group of ten, say, and the amount of compound interest earned in a given number of years or months – symbolic algebra frees mathematicians to generalise beyond the immediately obvious.

For example, taking the binomial expansion of $(1+x)^{\frac{1}{2}}$ gives a series expression for $\sqrt{1+x}$; approximate solutions can be found by taking a certain number of terms in the series, depending on the required accuracy. Isaac Newton proved the full binomial theorem, while Harriot had used binomial coefficients with fractional values of n in a related context: his pioneering formula for algebraic interpolation.

The use of series to represent or approximate functions, and interpolation to estimate values of functions, illustrates the kinds of mathematics needed to program computers and calculators to do algebra, and to solve an array of real world problems by hand or machine.

But none of it would have been possible without the abstract power of algebraic symbolism. And I haven't yet mentioned calculus – the greatest achievement during the explosion of mathematical creativity in the seventeenth century, when symbolic algebra took off.

Some students struggle with the abstractness of algebra, but history can help by showing how long it took for mathematicians to develop this way of thinking, and how simple problems can actually be easier with symbols than words. And going in a bit deeper, it can remind teachers how marvellous mathematics is!

Robyn Arianrhod's popular science books include *Thomas Harriot: A Life in Science*, and *Einstein's Heroes: Imagining the World Through the Language of Mathematics*.

HISTORY OF MATHEMATICS

Terence Mills

Several movies have been released that deal with aspects of the history of modern mathematics. By and large, they are reasonably accurate historically, and the acting is excellent. I will review them but not spoil them in case you have not seen them.

HIDDEN FIGURES (2016)

(Rated PG)

Hidden Figures is the story of Katherine Johnson B.S. (b. 1918), Dorothy Vaughan B.A. (1910-2008), and Mary Jackson B.S. (1921-2005), three African-American female mathematicians during the heady years of the space race in the US. Racial discrimination was part and parcel of every day life in the US at that time, and mathematicians were not immune from the consequences. The women portrayed in this movie had many battles that we can barely imagine now.

THE MAN WHO KNEW INFINITY (2015)

(Rated PG)

This movie deals with the life of the Indian mathematician Srinivasa Ramanujan FRS (1887-1920). Ramanujan was a self-taught mathematician who made many discoveries by himself in India. Eventually he made his way to the University of Cambridge where his talents were recognised for the world to see. His is a truly remarkable story.

THE IMITATION GAME (2014)

(Rated M)

This movie tells the story of Alan Turing PhD, OBE, FRS (1912-1954), the British mathematician who played a major role in breaking German codes during World War 2. Turing machines are named after him. The movie reminds us that mathematicians have contributed to war efforts, even since the days of Archimedes. In addition to the pressures of war, Turing experienced considerable personal and social pressures. The ending of the movie is sad and tragic.

CONNECTING CLASSROOM MATHS

Helen Haralambous - Mathematics education consultant, MAV

CONNECTING CLASSROOM MATHEMATICS WITH THE WORKPLACE THROUGH INDUSTRY BASED INVESTIGATIVE PROJECTS

Making connections between classroom mathematics and 'real world' applications supports students understanding of how mathematics is used in the workplace and enables students to see possible future careers that studying mathematics can take them.

For the past four years MAV has run a Maths Camp, in Melbourne for Year 10 rural and regional students, during the first week of the school holidays. The 24 students, selected via a competitive application process, were given the opportunity to gain insight into careers in the STEM industry.

The camp provided high potential students the opportunity to work with mathematicians and industry representatives, solving real-world maths problems in teams and expanding their mathematic skill sets.

The camp was made possible via Department of Education and Training funding and the MAV partnered with FORD, RBA, Texas Instruments, and Victorian Space Science Education Centre to create real world maths investigations in the fields of Engineering, Economics/Commerce, Coding and Biomedical research.

Students worked in groups of five and undertook problem solving activities as they worked through a scenario presented to them at the beginning of camp.

Students were supported and mentored by mathematicians from RMIT and La Trobe Universities.

The camp culminated with the students presenting their project findings to MAV industry partners, invited guests and parents who attended the presentations at La Trobe University on the final day.

Years before mission	Astronaut 1 BMD	Astronaut 2 BMD	Astronaut 3 BMD
0 (age 30)	1050	1500	1250
1			
2			
3			
4			
5			

Time on ISS (days)	Astronaut 1 BMD	Astronaut 2 BMD	Astronaut 3 BMD
0	1050	1500	1250
30			
45			
60			
75			
90			
105			
120			
135			
150			
165			
180			

Astronaut BMD data provided by VSSEC. In each case, assume start at age 30.
https://www.nasa.gov/mission_pages/station/research/benefits/bone_loss.html

THE 2019 PROJECTS INCLUDED

1. Exploring changes in the cash rate and its effect on the economy
2. Designing a snap fit clip on the exterior trim of the tailgate of a Ford Ranger
3. Using coding and engineering to construct a model car with a pet-smart alarm system.
4. Investigating the accuracy of Body Mass Index
5. Investigating bone mineral density in astronauts

As a celebration of the 50th anniversary of the first lunar walk, here is an investigation of Astronaut Bone Mineral Density.

INVESTIGATION BACKGROUND

The decrease in bone density during space travel and its effects on long term space deployment are of great concern to mission specialists. A normal human on earth experiences a 1% decrease in bone mineral density (BMD) per year from after the age of 30.

An astronaut in space may experience a BMD decrease of 1% per month!

STUDENT INVESTIGATION

Can you model the relationship between BMD readings on earth and their corresponding BMD readings from space?

How long would the astronauts returned to earth need to recover their BMD to original readings?

ADDITIONAL INFORMATION: ENABLING PROMPTS

Using the set of astronaut BMD data provided by VSSEC, you will need to create your own databases based on the measurements obtained from the initial data given.

This will require developing linear equations to describe the relationships and graphing the data developed.

Write down the general equation of a linear function.

EXTENSION OR ADVANCED INVESTIGATION

An astronaut travelling to Mars and back would be expected to be travelling through space for up to 2.5 years. Plot their BMD loss over this period of time.

It has been considered that sending older astronauts (50 years) to Mars might be a more advantageous proposition, as they will have accrued a great deal of experience and specialist training, as well as being less susceptible to certain types of cancers. Using the image (https://en.wikipedia.org/wiki/Osteoporosis#/media/File:615_Age_and_Bone_Mass.jpg), can you plot BMD for a male and female individual both on Earth and in space and make predictions of BMD loss in male and female astronauts of this age travelling to space?

2020 CAMP

Over the years MAV has been running the camp students, parents, academics and industry partners report on the stimulating, team and knowledge building experience for students.

As one student participant summed up the experience 'The camp is one of the best experiences I have participated in so far. As well as exposing me to an investigation that I would not be offered otherwise, it gave me an opportunity to experience activities that gave me an idea as to where maths can lead me in the future. I also was given the opportunity to make friends with people who have the same interests as me and can give me a different perspective on things I wouldn't have thought of otherwise. Thank you so much for providing me with a great experience that I will remember forever.'

MAV's 2020 Maths Camp will take place from Monday 29 June - Friday 3 July. Applications open on Tuesday 28 January. Visit www.mav.vic.edu.au for more information.

PUZZLES

Michael Nelson - Learning specialist, Drysdale Primary School

LOWER PRIMARY

I am a four digit number. My second digit is four times bigger than the third digit. My first digit is three less than my second digit. Who am I?

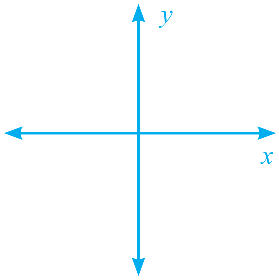
Group, partition and rearrange collections up to 1000 in hundreds, tens and ones to facilitate more efficient counting. (VCMNA105)

MIDDLE PRIMARY

Find two coins that when you double them equal 40 cents.

Represent money values in multiple ways and count the change required for simple transactions to the nearest five cents. (VCMNA137)

UPPER PRIMARY



Ari put one point in two quadrants of a Cartesian plane. When he drew the lines joining the dots, they formed a triangle. What could the coordinates be?

Introduce the Cartesian coordinate system using all four quadrants. (VCMMG230)

Which month has 27 days?

Use a calendar to identify the date and determine the number of days in each month. (VCMMG119)

I was cutting my pizza. As the numbers got bigger, my pieces got smaller. How is that possible?

Count by quarters, halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line. (VCMNA158)

A chicken was given \$9, an octopus was given \$36, a bee was given \$27. Based on this, how much did the cat get?

Multiply decimals by whole numbers and perform divisions by non-zero whole numbers where the results are terminating decimals, with and without digital technologies. (VCMNA215)

Images from Pixabay (L-R bottom row): Tigerlily713, Aline Ponce, Ernesto Rodriguez

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3. Select ***"Join"***
4. Complete the form and provide a copy of your school's current book list or a letter signed by your Head of Mathematics (or equivalent), verifying that your school booklists one or more of the CASIO models listed overpage
5. We will do the rest!

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http://shop.mav.vic.edu.au

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2019 MAV SOLUTIONS TO THE VCAA EXAMS

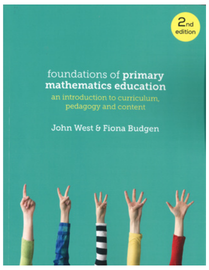
VCE

Pre-order for delivery in Term 1, 2020

Ensure students prepare effectively for VCE Mathematics examinations with the MAV solutions to the 2019 VCAA Examinations 1 & 2 for Mathematical Methods, Specialist Mathematics and Further Mathematics.

Each solution set includes: fully worked solutions for all sections, advice on solution processes, marking allocation and permission to reproduce copies for students.

INDIVIDUAL STUDY	$54.50 (MEMBER) $68.50 (NON-MEMBER)
ALL STUDIES	$127.50 (MEMBER) $159.50 (NON-MEMBER)



FOUNDATIONS OF PRIMARY MATHEMATICS EDUCATION

F-6

An introductory guide for pre-service primary teachers covering the principles of effective pedagogy and explaining the core content of the primary mathematics curriculum. Many pre-service teachers admit to feeling unsure about the mathematics they will have to teach. Others find it difficult to know how to apply the theories of teaching and learning they study in other courses to the teaching of maths. This book outlines key considerations of effective mathematics teaching and learning. Including understanding student motivation, classroom management, overcoming maths anxiety and developing a positive learning environment. Curriculum and assessment processes, and the use of ICT in the maths classroom are explored.

$46.48 (MEMBER)
$58.10 (NON MEMBER)

MAV SACS SUGGESTED STARTING POINTS 2020

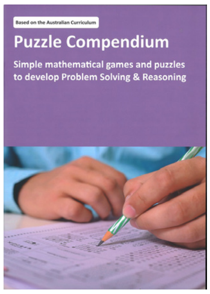
VCE

The MAV 2020 VCE Mathematics SACS materials are designed to provide suggested starting points for VCE Mathematics teachers for their School Assessed Coursework (SAC).

MAV SACS 2020 materials have been written by experienced VCE mathematics teachers. They are for use by teachers to aid in assessment of student School Assessed Coursework for Further, Methods and Specialist Mathematics.

PLEASE NOTE: this resource is only available to current practising Victorian secondary teachers. The product is for school use only. It is not to be used by private tutoring services.

INDIVIDUAL STUDY	$54.50 (MEMBER) $68.10 (NON MEMBER)
ALL STUDIES	$152.60 (MEMBER) $190.70 (NON MEMBER)



PUZZLE COMPENDIUM

4-9

Mathematical games and puzzles have been used as a source of amusement and entertainment for thousands of years. It is now recognised that puzzles and games represent an authentic context for the development of mathematical problem solving and reasoning skills. Students who develop these mathematical proficiencies of problem solving and reasoning can then apply them to a range of increasingly sophisticated mathematical problems.

This collection of 120 mathematical games and puzzles was compiled by Dr John West. It includes 17 different types of puzzle designed to provide an appropriate level of challenge for students of different ages and abilities.

$15 (MEMBER)
$18.75 (NON MEMBER)